PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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Phil. Trans. R. Soc. Lond. A 1987 **322**, 125-131
doi: 10.1098/rsta.1987.0042

Phil. Trans. R. Soc. Lond. A **322**, 125–131 (1987)
Printed in Great Britain

Plasma magnetic insulation

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Theoretically the strong magnetic field of a tokamak should confine electrons and ions in a high-temperature plasma sufficiently well. However, the behaviour of electrons turns out to be anomalous, especially with application of auxiliary heating. This behaviour is discussed in the framework of dimensional analysis and a hypothesis of plasma transition into a relaxed state with a minimal energy subjected to the single constraint, namely conservation of the total current. Plasma tends to relax to the optimal current-density and pressure profiles, any deviation from which results in a additional heat transfer via the micronoise pumping by the magnetic island structure. This effect is more pronounced with an increase of n/I_p, where n is the density, I_p is the plasma current.

Introduction

The magnetic field of the tokamak consists of the strong toroidal field B_T and a weaker poloidal field B_θ produced by plasma current. The magnetic fields B_T, B_θ, together with external fields, provide an equilibrium, macroscopic plasma stability and a rather good confinement of ions and electrons. This statement refers, first of all, to ions; their almost neoclassical confinement has been observed in many facilities. As for electrons, their behaviour has always been anomalous, and the statement of their sufficiently good confinement only means that it remains acceptable for controlled fusion applications.

Much effort has been spent on finding empirical laws for the electron heat conduction and plasma diffusion. For the ohmic régime this seemed to be completed successfully by the T-11 scaling law and by the neo-ALCATOR one. However, a degradation in the confinement takes place when auxiliary heating is used.

Goldston (1984) succeeded in finding an empirical relation for the energy-confinement time that covers both the ohmic- and auxiliary-heating results. Then the electron loss channel sprang an additional surprise, the so-called profile consistency. Coppi (1980) was the first who paid attention to the phenomenon of profile consistency. A heat-transfer sensitivity to the temperature and plasma density profiles was first observed in small and medium-sized devices. Later, the phenomenon of profile consistency, i.e. temperature profile insensitivity to the power profile deposition, was clearly demonstrated in large machines, TFTR (Furth *et al.* 1985) and JET (Bickerton 1986).

A series of experiments with radiofrequency heating at the electron-cyclotron resonance frequency (ECRH), allowing localized power deposition into various plasma column regions, was made on T-10 (T-10 Group 1985) to study this phenomenon in detail. These experiments have shown that the phenomenon of optimal profile conservation is manifested more effectively at high values of n_e/I_p, where n_e is the electron density, I_p is the plasma current. If the power deposition does not deviate too much from the optimal profile, the confinement corresponds to that of the ohmic régime; in the opposite case, a degradation in confinement is observed.

How to understand all these phenomena? This is the subject of the paper.

Dimensional analysis

Dimensional analysis turns out to be very convenient for experimental data processing; it allows one not only to reduce the number of variables but, in addition, to select the most important of them. This statement has been confirmed many times in the physics of fluids, gases and ionized gases. So let us apply dimensional analysis to the tokamak plasma (Kadomstsev 1975; Connor & Taylor 1977). Here, as in many other cases, success depends on the proper choice of dimensionless variables. One can show that some variables are not so important for the tokamak electrons: mass ratio, m_e/m_i; collisionality parameter of the neoclassical theory, ν_*; Langmuir number. The important parameters are, first of all, the geometrical ones:

$$A = R/a; \quad q_a = B_T a/B_\theta R, \tag{1}$$

where R is the major radius of the torus, a is the minor radius of plasma, q_a is the safety factor at the plasma edge. Then two additional parameters are important:

$$\beta = 2\mu_0 \bar{p}/B^2, \quad \Pi_e = \pi r_e a^2 n_e. \tag{2}$$

Here, β is the ratio of the plasma pressure $\bar{p}$ to the pressure of magnetic field; Π_e is the linear electron density, $r_e = e^2/m_e c^2$ is the classical electron radius.

Besides q_a, we shall use the local value of $q = B_T r/B_\theta R$ and its reciprocal quantity $\mu = 1/q$, which has the sense of a rotational transform angle divided by 2π.

The dimensional quantities can be arranged as products of a certain dimensional quantity and a function of dimensionless variables. For example, the energy-confinement time in the ohmic régime, $\tau_{E,oh}$, turns out to be proportional to the product of the transit time $q_a R/v_{th}$ (v_{th} is the thermal velocity of electrons) and some factor:

$$\tau_{E,oh} \sim A^2 \Pi_e^2 q_a R/v_{th} \tag{3}$$

This is the so-called T-11 scaling law. The expression for $\tau_{E,aux}$ is somewhat more complicated.

Sawtooth oscillations

Later we shall discuss the mechanism of profile self-consistency. Therefore, it is natural to begin with the simplest example of the phenomenon of such a type: the current density limitation near the plasma column axis by the sawtooth oscillations. The sawtooth oscillations are observed in many tokamaks. In JET, for example, they result in very large temperature oscillations near the central region under auxiliary heating. The sawtooth oscillations are caused by the development of an $m = 1$ tearing mode. The tearing-mode radial dependence for a perturbation of the magnetic flux, satisfying the necessary boundary conditions at the plasma column centre and on the casing ($r = b$), is outlined in figure 1. If the perturbations converge at a point where $q = 1$ as is shown in figure 1*a*, the tearing mode will be stable, and, vice versa, when $\Delta' > 0$, as in figure 1*b*, the tearing instability will develop. For a slow development of the $m = 1$ mode, one can expect the formation of an island, figure 2*a*. A newly formed island will push the central part to the reconnection point.

However, one should not forget that the field-line reconnection process is not a quiet one; field lines are stretched away from the reconnection point. Therefore, the second version of reconnection, figure 2*b*, is quite probable when the whole area of an island is filled with the

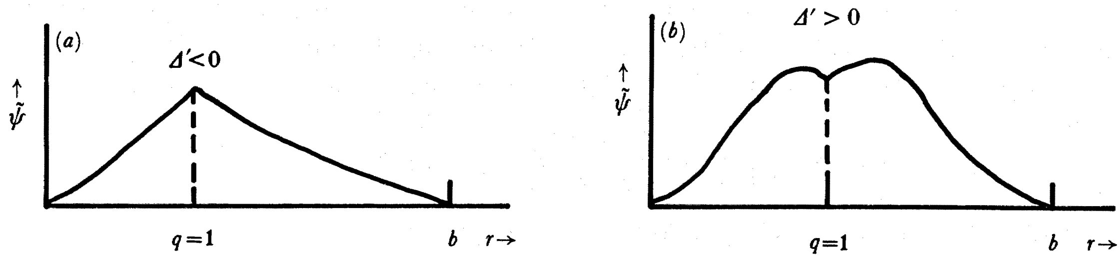

FIGURE 1. Radial dependences of the magnetic flux perturbation for the stable, $\Delta' < 0$, and unstable, $\Delta' > 0$, tearing modes.

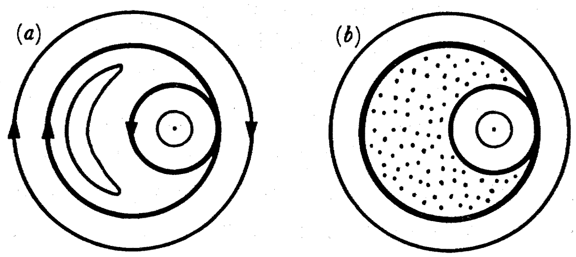

FIGURE 2. Structure of the quiescent magnetic island (*a*) and stochastized one (*b*) for the $m = 1$ mode.

'boiling', stochastized field. In this case, the reconnection can be incomplete, such reconnections are also observed in many tokamaks. Sometimes, complete and incomplete reconnections interlace with each other, and the whole picture looks like double sawteeth. One should pay attention to one peculiarity of sawtooth oscillations. The sawteeth, either complete or incomplete, flatten the current density distribution near $q = 1$; and at the same time broaden the region where $q = 1$.

The tearing modes can develop for other combinations of m/n; in particular, the $(m = 2)/(n = 1)$ mode can develop near $q = 2$. The higher-order modes 'ring' for some time and also look like relaxation oscillations.

PROFILE CONSISTENCY

The temperature and current density profiles in tokamaks are self-organized, and, as has been noticed long ago, they are close to a universal bell-shaped dependence. Plasmas retain these profiles even under auxiliary heating, when the power deposition strongly differs from the ohmic one. It is clear that this fact is not an occasional one, it manifests a property of tokamak plasma self-organization when the transport processes are coupled by feedback with the self-arranged profiles in the plasma.

It would be natural to associate the optimal current profile with the tearing mode, as was emphasized by Furth *et al.* (1985), who has paid attention to the fact that the values of Δ' for the lowest modes are weakly positive for real profiles. Keeping in mind the elegant theory by Taylor (1974) on relaxed states in the reversed field pinch, it would be desirable to develop a similar approach. Taylor's theory in its direct form cannot be applied to tokamak. The point is that it assumes complete field-line reconnections. This situation only appears in tokamak during disruption. In addition to this, the energy of the longitudinal magnetic field in a tokamak enclosed in a shell conserving longitudinal flux remains unchanged, and the poloidal-field energy and the thermal energy of plasma are often comparable with each other. Therefore, the energy principle should be modified. It can be represented in a form

$$\delta \mathrm{F} = \delta\left(\int \frac{B_\theta^2}{2}\, r\,\mathrm{d}r + \frac{1}{\gamma-1}\int pr\,\mathrm{d}r + \lambda \int jr\,\mathrm{d}r\right) = 0, \tag{4}$$

where $p(\mu)$ is the plasma pressure, $j(\mu)$ is the current density.

In other words, the statement is as follows: a relaxed state with the minimal total energy (that of the poloidal magnetic field plus thermal energy) at a given current I_p exists. The quantity λ should be considered as a Lagrangian multiplier, the expression in brackets can be interpreted as the free energy subjected to the constraint $I_\mathrm{p} = \mathrm{const}$.

Varying the expression in brackets, one should keep in mind that we would like to restrict ourselves to the tearing modes. This means that we should consider states close to the initial one, with variations for which the current density distribution, $\mu = 1/q$ and p near the islands, respectively, become more or less flat, with conservation of the magnetic flux at some distance from the islands. This means that we should assume $B_\theta = \mathrm{d}\psi/\mathrm{d}r$, where ψ is the magnetic flux of the poloidal magnetic field. Moreover, it is natural to assume that $p = p(\mu)$; $j = j(\mu)$. The last assumption exactly means that the island structure formation is accompanied by a simultaneous profile flattening of j, μ, p in the vicinity of the initial configuration.

Thus, varying the expression in brackets with respect to ψ and taking account of

$$\mu = \frac{R}{B_\mathrm{T}}\frac{1}{r}\frac{\mathrm{d}\psi}{\mathrm{d}r},$$

one obtains

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{B_\mathrm{T}}{R}\frac{\mathrm{d}\psi}{\mathrm{d}r} + \frac{1}{\gamma-1}\frac{\mathrm{d}p}{\mathrm{d}\mu} + \lambda\frac{\mathrm{d}j}{\mathrm{d}\mu}\right) = 0. \tag{5}$$

One can see that the first term equals $(B_\mathrm{T}^2/R^2)r^2\mu$. It tends to a constant as $r \rightarrow \infty$, i.e. $\mu \rightarrow 0$.

Let us consider first the simplest solution of (5), namely $p = p_0\mu^2$, $j = j_0\mu^2$, where p_0 and j_0 are constants. Then, from (5) one finds

$$\mu = (1 + r^2/a_*^2)^{-1}, \tag{6}$$

where the integration constant is chosen so that $\mu(0) = 1$, although it is not obligatory (but corresponds to the majority of tokamaks). Accordingly, the pressure and current density distributions have a form:

$$p = p_0(1 + r^2/a_*^2)^{-2}; \quad j = j_0(1 + r^2/a_*^2)^{-2}. \tag{7}$$

Note that the distribution (7) for $j(r)$ was obtained previously by D. Biskamp (personal communication 1986) from a similar variational principle, assuming that $p = 0$, $j = j(\psi)$. Now let us take into account the relation $j/j_0 = (1/2r)\,(\mathrm{d}/\mathrm{d}r)\,(r^2\mu)$. One can see that the solution (7) is consistent with this expression for j/j_0. One can show that the partial solution chosen by us is, at the same time, a unique one (among the reasonable solutions).

Thus, the model of relaxed state results in universal profiles:

$$q = 1 + \rho^2; \quad p = p_0(1 + \rho^2)^{-2}; \quad j = j_0(1 + \rho^2)^{-2}, \tag{8}$$

where $\rho = r/a_*$ is the dimensionless variable, and the quantity $a_*^2 = I_\mathrm{p}/\pi j_0$.

When $q(0) = \mu(0) = 1$, this quantity is given by

$$a_*^2 = R^2 I_p / I_0 = R I_p / 5 B_T, \tag{9}$$

where I_0 is the total number of Ampère-turns in the coils of the toroidal magnetic field.

As the current density in tokamak is proportional to $T_e^{\frac{3}{2}}$ and the pressure and current density distributions are similar to each other, the density $n \sim \surd T_e$ in relaxed state.

Let us consider again the expression for free energy (4), which serves us for formulation of the variational principle. We can eliminate the Lagrangian factor, substituting the value that follows from (6) for it. Then one obtains an expression for free energy at the relaxed state:

$$F = \frac{B_T^2}{2R^2} \int (r^2 + a_*^2)\, \mu^2 r\, \mathrm{d}r. \tag{10}$$

One can see that the thermal energy is eliminated from this expression; this results in certain consequences discussed below.

Profile relaxation

The mechanism of direct, anomalously fast, magnetic field dissipation is practically absent in the tokamak plasma (except the current disruptions). Therefore, a rather complicated chain of couplings among relaxational mechanisms emerges.

The tearing modes originating from the poloidal magnetic field energy can also be somehow 'pushed' by plasma; that is taken into account by the second term in the total energy relation (4). Its ratio to the first one is proportional to β_p, the ratio of plasma pressure to the poloidal field pressure. However, the tearing modes weakly affect the global magnetic field; they give birth to island structures only. The islands, in their turn, may result in the plasma energy dissipation. The islands, moving with respect to each other, 'stir' the magnetic structure and thus provide magnetic noise pumping into the plasma. This results in the anomalous electron heat conduction and diffusion.

As one can see from (10), the free energy at its minimum does not depend on the thermal-plasma energy. Therefore, the relaxed state with satisfactory confinement can exist at any level of heat energy in plasma. However, the free energy can start to act for a deviation from the optimum: the greater β_p, the more effective it will be. This conclusion is well correlated with the experimental results from T-10 on ECRH. The consequences of heating for régimes with different n/I_p were studied in those experiments. In all régimes the distributions were close to the self-consistent ones, their profiles were similar to (8). As the temperature was about the same, the parameter

$$n/I_p \sim n T a_*^2 / I_p^2 \sim \beta_p.$$

The experiment has shown that the sustainment of an optimal profile becomes more pronounced with an increase in $n/I_p \sim \beta_p$; the plasma tries to retain it. If the power deposition is made very carefully, i.e. conserving a profile close to the ohmic one, inside the region $q < 2$, the ohmic scaling law is well satisfied. On the contrary, the non-central heating, which probably flattens the current-density distribution at the periphery, results in the confinement degradation which is developed for a skin time. The relevant confinement time corresponds to the Goldston scaling law for $\tau_{E,\,aux}$. The régime with maximal n/I_p on T-10 is of a particular interest: the plasma confinement degradation is revealed in this régime in the easiest way, for instance, by the shift of the limiter to the plasma edge.

Energy confinement

The mechanism of profile consistency results in an extraordinary phenomenon in the tokamak plasma. The heat-conduction profile readjusts itself to the auxiliary power profile $P(r)$. Under strong disturbance of the profiles, their fast self-adjustment occurs; e.g. fast redistribution of profiles in ALCATOR-C under pellet injection and that in TFTR under adiabatic plasma compression. The enhanced heat conduction of the heat pulse propagation from sawtooth oscillations is probably related to the same range of phenomena.

At the same time, if one speaks about the global energy confinement, which is characterized by the energy confinement time τ_E, it reacts more softly to the profiles, including the energy deposition profile. The general picture of scaling laws for τ_E is rather successfully described by Goldston's relation:

$$\tau_E^{-2} = \tau_{E,\,oh}^{-2} + \tau_{E,\,aux}^{-2}, \tag{11}$$

where $\tau_{E,\,oh}$ and $\tau_{E,\,aux}$ describe the régimes with low and high densities.

The electron heat conduction, produced by the magnetic noise pumping (Kadomtsev 1985), does not depend on the level of pumping at low density. That is why an empirical dependence for electron thermal conductivity K_e is possible.

At high densities electron transport becomes sensitive to the level of pumping, i.e. to the power deposition profile and to the current density profile, respectively. It also becomes dependent

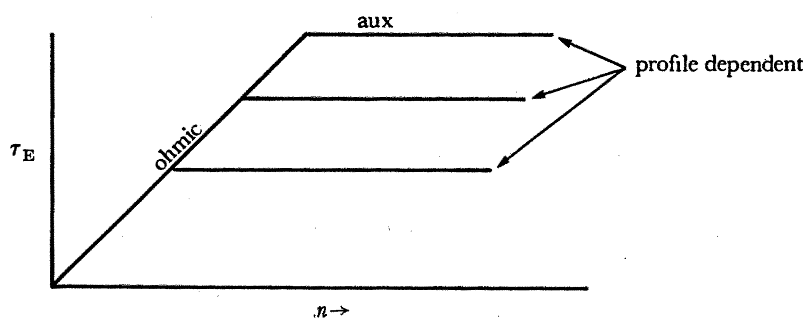

FIGURE 3. The energy-confinement time against density: at low n, the ohmic-scaling law is true; at high n, the scaling law for the auxiliary heating is valid. The quantity $\tau_{E,\,aux}$ is sensitive to the current-density and power-deposition profiles.

on the profiles of current and power deposition (see figure 3). This effect also takes place with the ohmic heating in an implicit form; the auxiliary heating allows us to see it more distinctly. The transition from one régime of confinement to another occurs at lower densities in large machines as compared with the medium-size facilities. For example, this transition in JET occurs at about $n_e = 3 \times 10^{19}$ m^{-3}.

Conclusion

At present, the physics of plasma electron confinement is qualitatively rather clear. The heat transfer is controlled by a weak stochastization of the magnetic field. At low density this results in the T-11 scaling law or the neo-ALCATOR one. In this case the local-diffusion and electron-heat conduction coefficients have quite a definite sense.

In another limiting case, when β reaches its critical value β_c with the respect to the ballooning modes, the plasma pressure profile is fixed, and further increase in the power deposition into plasma does not result in a rise in its energy.

The most interesting range is in between these two limiting cases. One can propose an idea of a relaxed state with minimal total energy (thermal plus the poloidal-field energy) at a given total plasma current with respect to the tearing modes, which control the plasma transport in this range. If the power deposition slightly disturbs the profiles of a relaxed state, one will be able to extend the ohmic-heating scaling law sufficiently far with density increase.

Nevertheless, sooner or later the transition to the scaling law $\tau_{E,\,aux}$ takes place. The less optimal the temperature and current density profiles are, the earlier this transition takes place. In this case, the higher the value of β_p, the more actively the plasma resists any deviation from the optimal profiles. First, a self-adjustment of heat conduction to the power deposition profile takes place in the range of moderate β_p. Second, there is a certain possibility of increasing τ_E by power deposition profile optimization. One should hope that further studies will allow the formulation of the quantitative relations for the relaxed state.

References

Bickerton, R. J. 1986 In *Proc. 12th European Conference on Controlled Fusion and Plasma Physics, Budapest, 2–6 September 1985*. (*Plasma Phys. controlled Fusion* **28**, 55–70.)
Connor, J. W. & Taylor, J. B. 1977 *Nucl. Fus.* **17**, 1047–1056.
Coppi, B. 1980 Comments Plasma Phys. controlled Fusion **5**, 261.
Furth, H. P. *et al.* 1985 In *Proc. 12th European Conference on Controlled Fusion and Plasma Physics, Budapest, 2–6 September 1985*. (*Europhysics conference abstracts* **9F** (2), 358–361.)
Goldston, R. 1984 *Plasma Phys. controlled nuclear Fusion* **26**, 87–103.
Kadomtsev, B. B. 1975 *Fiz. Plasmy* **1**, 531–535. (*Soviet Phys. J. Plasma Phys.* **1**, 295–299.)
Kadomtsev, B. B. 1985 *Comments Plasma Phys. controlled Fusion* **9**, 227–237.
Merezhkin, V. G. & Mukhovatov, V. S. 1981 *Soviet Phys. JETP Lett.* **33**, 463–467.
Taylor, J. B. 1974 *Phys. Rev. Lett.* **33**, 1139–1141.
T-10 Group 1985 In *Proc. 12th European Conference on Controlled Fusion and Plasma Physics, Budapest, 2–6 September 1985*. (*Europhysics conference abstracts* **9F** (1), 42–45.)

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